A quick self test for Environmental Physics at the University of Bremen

Bremen, May 2015

For the PEP master course, we work intensively with experimental and theoretical concepts of the atmosphere, ocean, ice, and climate system. In order to check if you have the necessary mathematical and physical background, you can make this self test to check if you are suited for our programme.

1. Given
$$f(x, y, z, t) = x^2 + y^2 + z^2 \sin(\omega t).$$
What are the partial derivatives with respect to the variables x and t?

2. What is the definition of ∇, Laplace, divergence, total (substantial) derivative, total differential for a function $f(x, y, z, t)$?

3. Calculate the rotation of ∇f.

4. Given the function $g(x) = ax^2 - 3x^4 + 2x \sin(\alpha x)$, please provide the Taylor expansion of g around $x = 0$ up to the 3rd order in x !

5. In the atmosphere, ocean, ice system, we are dealing with forces. Please list some relevant real and apparent forces.

6. What is the differential equation describing radioactive decay? Please provide also the solution with initial condition $x(t = 0) = x_0$. How is the half-life time defined?

7. The highest biulding on the campus of the University of Bremen is the so-called drop tower (see Fig. 1 upper panel). How far one can look onto the horizon under good weather conditions? Denote this distance by d. Remember the Earth's radius $R = 6378km$, the tower is h=110 metres high, and apply Pythogoras.

8. Related to the last question: Why is the rule-of-thumb $d = \sqrt{2hR}$ a good approximation? (For h=10m this means d=11 km.)
When h is in m, d in km, the formula can be written as $d = 3.5\sqrt{\frac{h}{\mathrm{m}}}$ km.

9. The town Bremerhaven which is at the North Sea is about 60 km north of Bremen (see Fig. 1 lower panel). How big must a tower in Bremen be in order

	Horizontal Length L	Velocity V	Time T
Microturbulence	1-10 cm	1-10 cm/s	seconds
Thunderstorms	1-10 km	10 m/s	hours
Weather patterns	100-1000 km	1-10 m/s	days to weeks
Climatic variations	global	1-10 m/s	decades and beyond

Table 1: Table shows the typical scales in the environmental, atmosphere, ocean and climate system. Using these orders of magnitude, one can derive estimates of the timescales.

to see the coast in Bremerhaven? (clear sky conditions, which is not always the case)

10. The drop tower in Bremen is used for experiments under conditions of weightlessness, which are otherwise only possible in space. That saves the high costs of a space mission. A capsule is raised to the top of the 110-metre-tall tower and released. How long does it take an object to fall down (vacuum conditions)?

11. Table 1 lists typical velocity, length and time scales of some fluid processes and systems. Not surprisingly, larger systems evolve on longer time scales. Depending on the size of the system under consideration, the spatial scale can be regional, continental or even global. Using the length and velocity scales (L and V), determine a typical time scale (T=L/V)! (Rough estimates are given in the last column in Table 1.)

12. As in 11: From the weather chart (Figure 2), identify the horizontal extent of a major atmospheric sea level pressure and the associated wind speed. Determine a typical time scale T !

13. The potential temperature of a parcel of fluid at pressure p is the temperature that the parcel would acquire if adiabatically brought to a standard reference pressure p_0, usually 100 kPa. The potential temperature of air is often given by

$$\Theta = T\,(p_0/p)^{R/c_p}$$

where T is the current absolute temperature of the parcel, R is the gas constant of air, and c_p is the specific heat capacity at a constant pressure. $\kappa = R/c_p = 2/7$ for an ideal diatomic gas. For a constant lapse rate $\frac{dT}{dz} = \gamma = const.$, why does the potential temperature Θ increase with height? Hint: Atmospheric pressure decreases with height.

Figure 1: Upper panel: Drop tower in Bremen. Lower panel: Harbor in Bremerhaven, ca. 60 km north of Bremen.

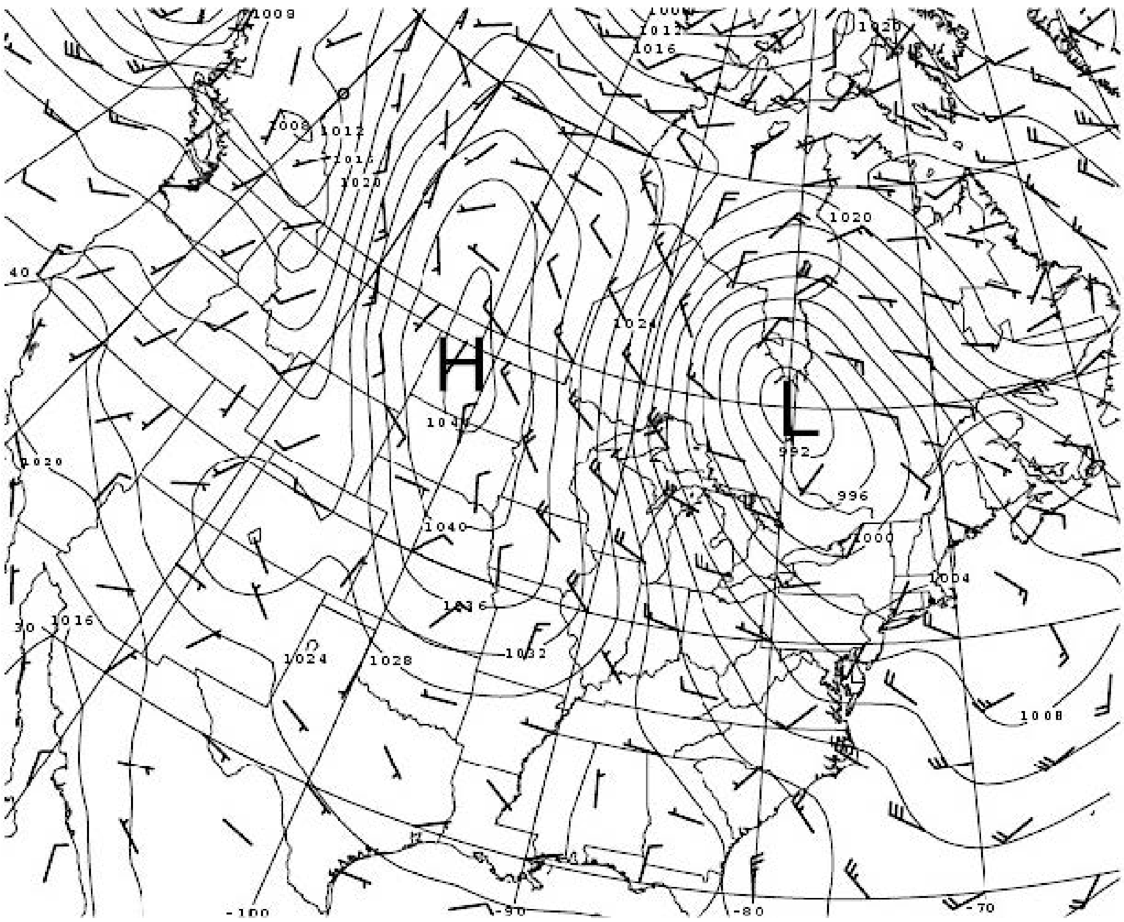

Figure 2: Surface pressure field and surface wind at 12GMT on 10th Feb, 2008. The contour interval is 4mbar. High and low pressure systems are marked. The dark segments represent wind arrows, whose arrowhead is not drawn in meteorological plots, by convention. The reader should imagine arrowhead at the end of segment that has no quivers. The quivers are drawn at only one side, at the tail end. The wind blows in the direction of the quiver base to the arrowhead. One full quiver represents a wind of 5m/s. We see air circling in a generally anticlockwise direction around the low but spiraling inwards, and air circling in a generally clockwise direction around the high but spiraling outwards.